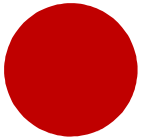
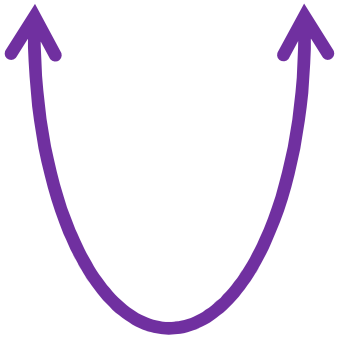
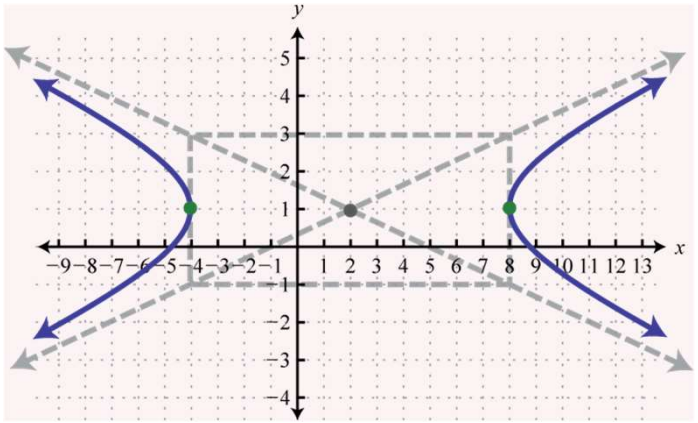
Ch. 11 Conics



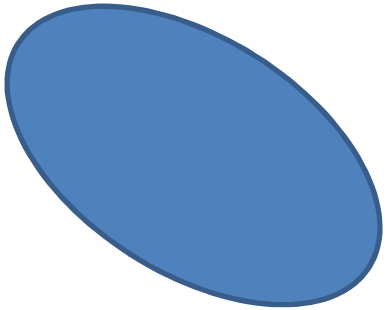
Circle



Parabola



Ellipse



Hyperbola

Add notes to pink sheet as needed:

Polar Coordinates

$r^2 = x^2 + y^2$ or $r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

polar form of a complex number

$r(\cos\theta + i\sin\theta)$

$z_1 \cdot z_2 =$

$r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

DeMoivre's Theorem

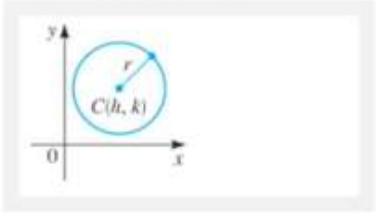
$[r(\cos\theta + i\sin\theta)]^n$

$= r^n(\cos n\theta + i\sin n\theta)$

Conic Sections

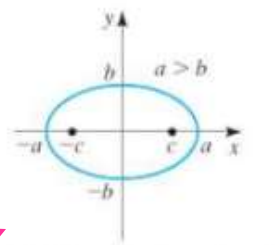
Circles

$(x-h)^2 + (y-k)^2 = r^2$



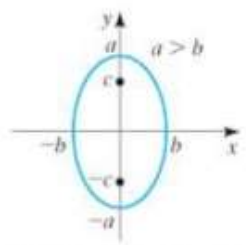
Ellipses

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

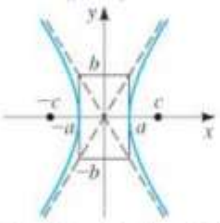
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$



Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

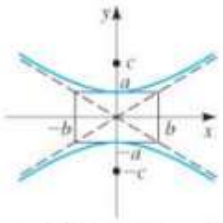
Hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



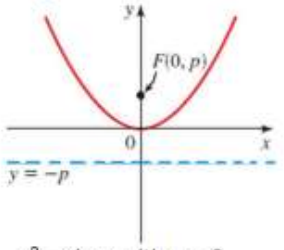
Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

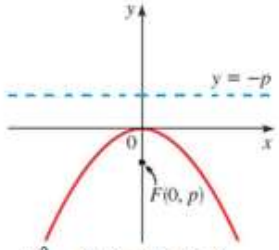


Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

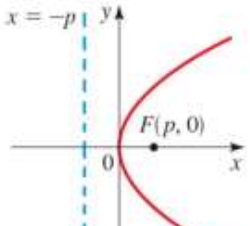
Equations and Graphs of Parabolas



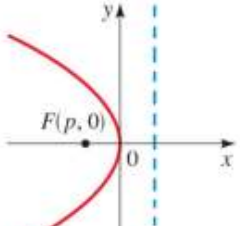
$x^2 = 4py$ with $p > 0$



$x^2 = 4py$ with $p < 0$

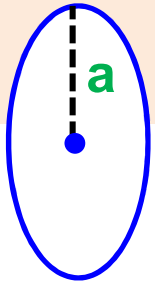


$y^2 = 4px$ with $p > 0$

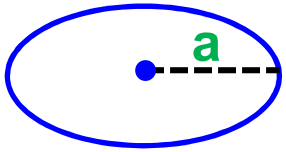


$y^2 = 4px$ with $p < 0$

See **pink sheet** for standard equation of an ellipse centered at (0, 0)



horizontal



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

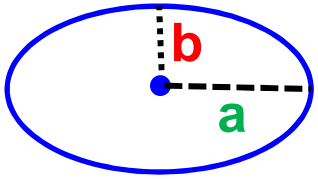
vertical

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

a^2 is the LARGEST value that creates the MAJOR axis = 2a

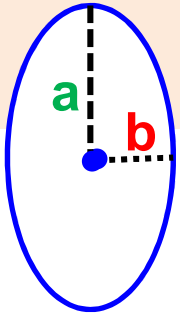
See **pink sheet** for standard equation of an ellipse centered at (0, 0)

horizontal

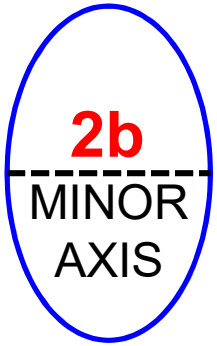


$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

vertical



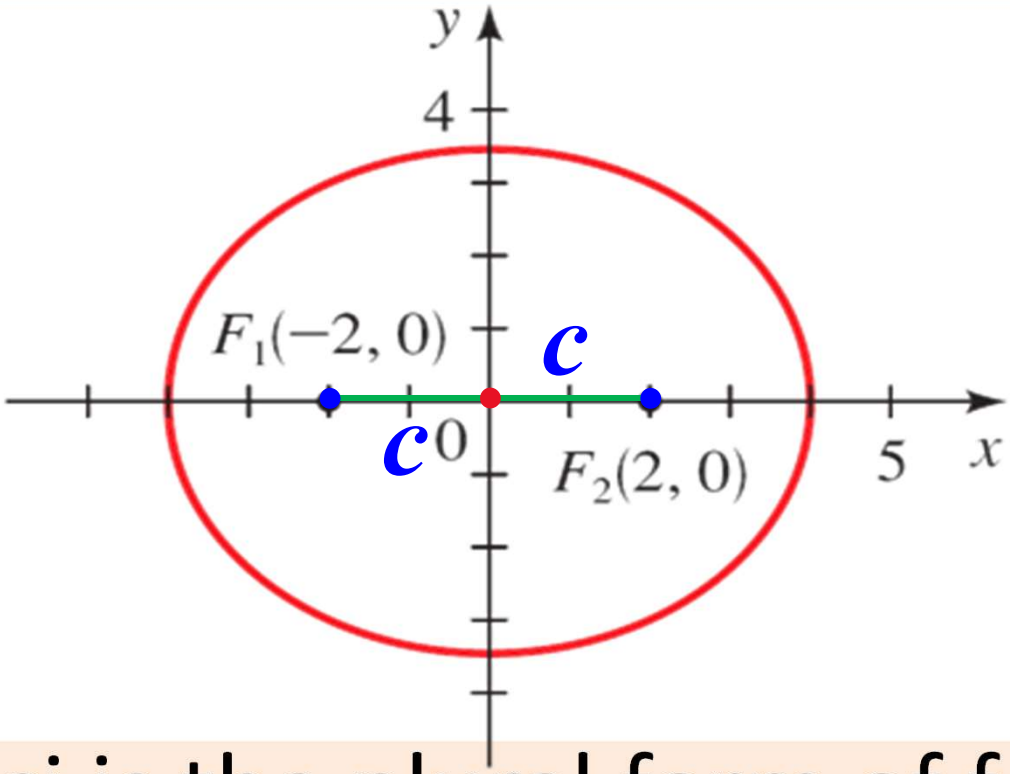
$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$



b^2 is the SMALLEST value that creates the MINOR axis = 2b

An ellipse has **2** focus points called the **foci** located "**c**" units from the center.

$$c^2 = a^2 - b^2$$

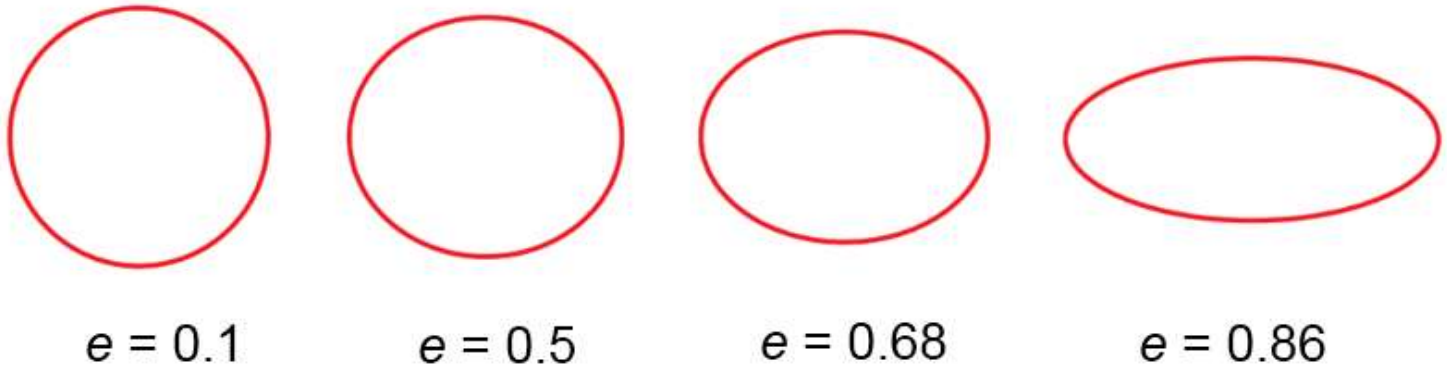


Note: foci is the plural form of focus

"soft" c sound *versus* *"hard" c sound*

(similar to vertices vs vertex)

The eccentricity is a measure of how "stretched" the ellipse is. $e = \frac{c}{a}$

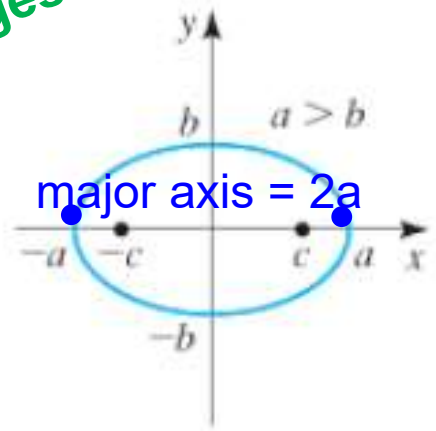


more circular **more elliptical (elongated)**

Eccentricity: $e = \frac{c}{a}$ **Ellipses**

horizontal $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

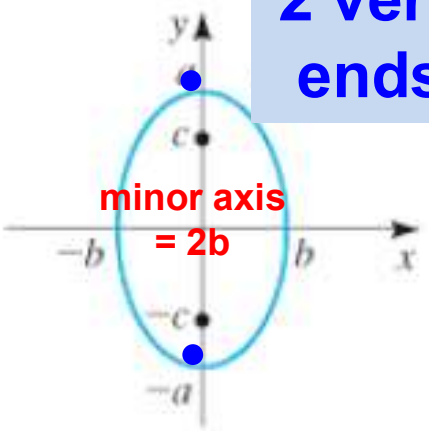
largest →



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

vertical

$\frac{x^2}{b^2} + \frac{y^2}{a^2}$ ← largest



Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

2 vertices always at ends of major axis

$$c^2 = a^2 - b^2$$

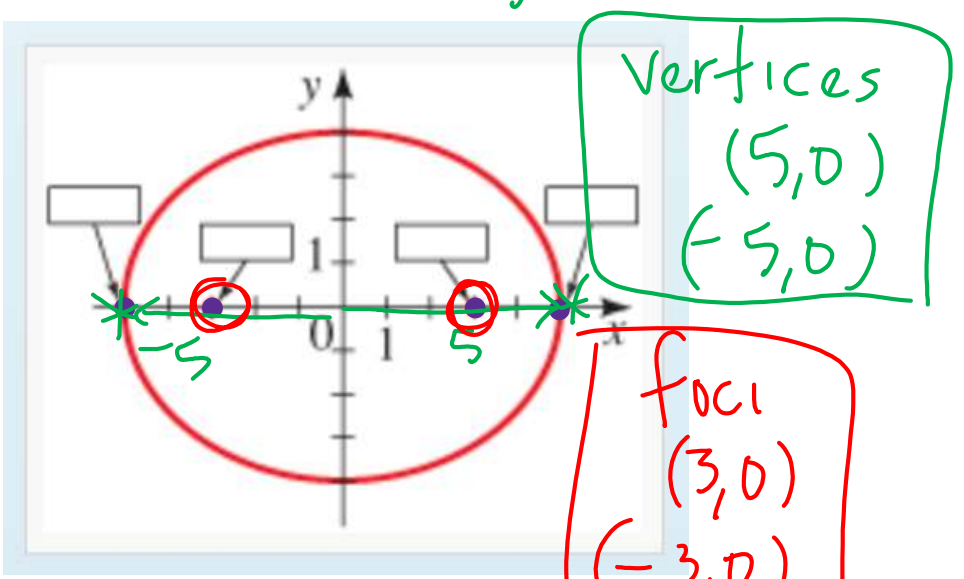
2 foci located on major axis

"c" units from the center

Add notes to pink sheet as needed

Label the vertices and foci for the given graphs:

4. (a) $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$ largest



Vertices
(5, 0)
(−5, 0)

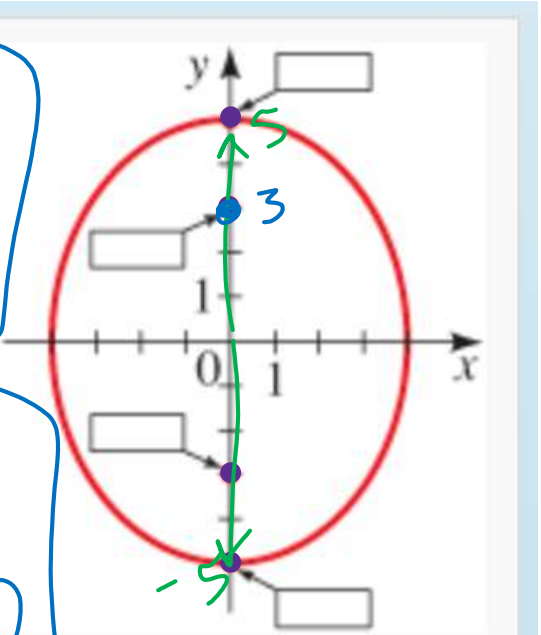
foci
(3, 0)
(−3, 0)

or $(\pm 3, 0)$

(b) $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$ largest

Vertices
(0, 5)
(0, −5)

foci
(0, 3)
(0, −3)



#5-8: Match the equations with the graphs

(write equation, show work!)

II

5. $\frac{x^2}{16} + \frac{y^2}{4} = 1$ horizontal

largest a^2 $\quad b^2$ $\quad a=4$ $\quad b=2$

IV

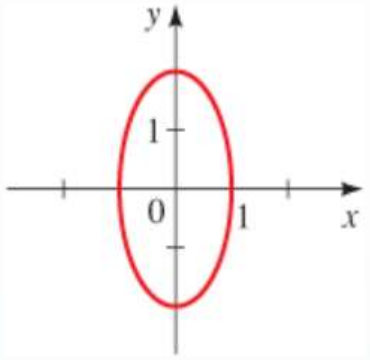
6. $\frac{x^2}{1} + \frac{y^2}{9} = 1$ vertical

b^2 $\quad$ largest a^2 $\quad a=3$ $\quad b=1$

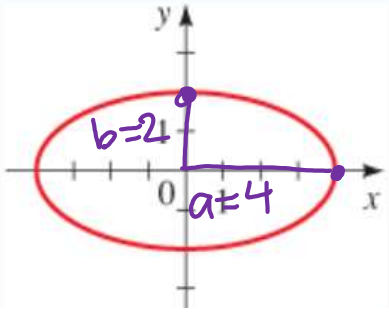
7. $4x^2 + y^2 = 4$

8. $16x^2 + 25y^2 = 400$

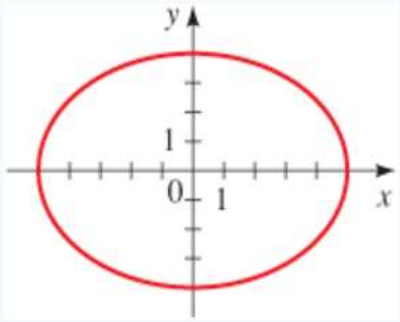
I.



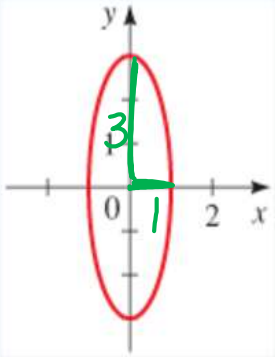
II.



III.



IV.



#5-8: Match the equations with the graphs

(write equation, show work!)

5. $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

6. $x^2 + \dfrac{y^2}{9} = 1$

7. $4x^2 + y^2 = 4$

I

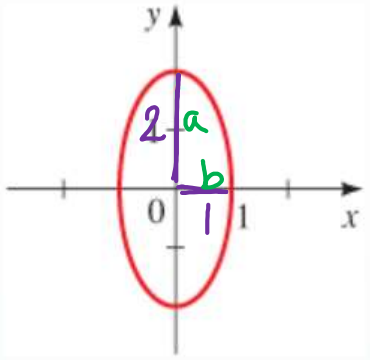
8. $16x^2 + 25y^2 = 400$

III

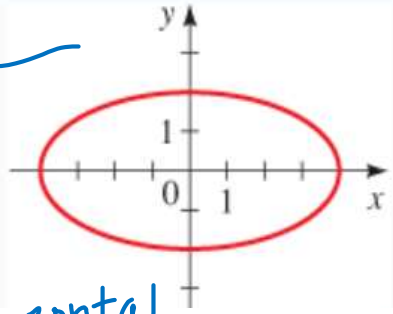
$\frac{16}{400}x^2 + \frac{25}{400}y^2 = \frac{400}{400}$

$25 \quad 16$

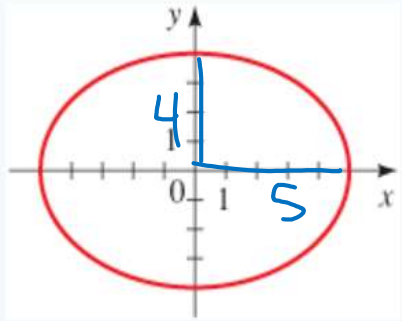
I.



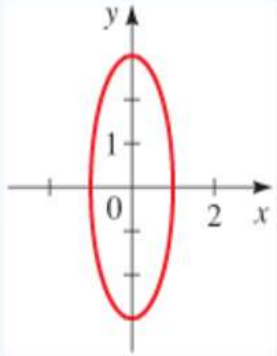
II.



III.



IV.



horizontal

$\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

a^2 (25) largest

b^2 (16)

$a = 5$

$b = 4$

#11-21odd

major $= 2a = 2(9) =$ 18

minor $= 2b = 2(6) =$ 12

(a) Find the vertices, foci, eccentricity.

(b) Determine lengths of major and minor axes.

(c) Sketch

vertical

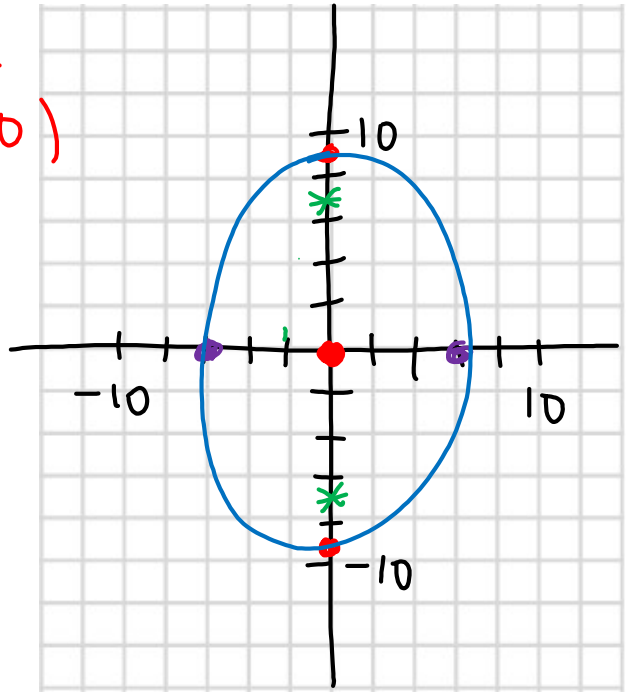
11. $\dfrac{x^2}{36} + \dfrac{y^2}{81} = 1$

center (0,0)

$36 = b^2$ $\quad$ $81 = a^2$ largest

$b = \sqrt{36}$ $\quad$ $a = \sqrt{81}$

$b = 6$ $\quad$ $a = 9$



graph first, then identify vertices
foci
eccentricity

see next slide

#11-21odd

(a) Find the vertices, foci, eccentricity.

(b) Determine lengths of major and minor axes.

(c) Sketch

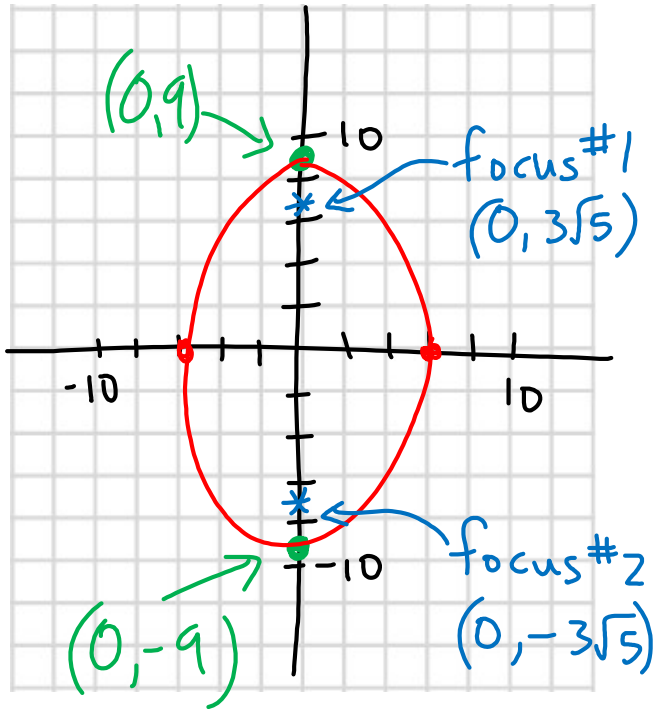
$$11.\ \frac{x^2}{36}+\frac{y^2}{81}=1$$

b^2 $\quad$ a^2

$b=6$ $\quad$ $a=9$

$e=\frac{c}{a} \rightarrow e=\frac{3\sqrt{5}}{9}$

so $e=\frac{\sqrt{5}}{3}$



(a) Vertices on major axis: $(0,9)$, $(0,-9)$

foci: $(0,3\sqrt{5})$, $(0,-3\sqrt{5})$

$c^2=a^2-b^2$

$c^2=9^2-6^2$

$c^2=81-36$

$c^2=45 \rightarrow c=\sqrt{45}=3\sqrt{5}$ ($\sqrt{9}\sqrt{5}$)

approx ≈ 6.6 (use calculator)